New Results on the Queens_n^2 Graphs Coloring Problem

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Given an $n \times n$ chess board, a queen graph is a graph with n^2 vertices, each corresponding to a square of the board. Two vertices are connected by an edge if the corresponding squares are in the same row, column, or diagonals (*both ascending and descending diagonals*), this corresponds to the queen move rule at the chess game. The coloring problem on this graph consists in finding the minimum number of colors necessary for placing n^2 queens on the board so that no two queens of the same color can attack each other. Finding this number (*the chromatic number* χ) is an optimization problem. We may also consider the following decision problem: given a n^2 chess board, is it possible to place n sets (*each corresponding to a given color*) of n queens on the board so that there is no clash between two queens in the same set? Gardner [3] states without proof that this is the case if and only if n is not divisible by either 2 or 3. If so, $n = \chi(\text{Queens\_}n^2)$ *–noted* χ_n– since maximum clique number is n. Until now, no chromatic numbers are available for $n > 9$ if n is a multiple of 2 or 3 (*see:* [1, 2, 4] *for recent works*).

In this talk, we propose an exact algorithm for solving the Queens_n^2 coloring problems. This algorithm exploits strongly a particular characteristic of these graphs: only independent sets with n vertices are useful to answer the question: *"is Queens_n^2 chromatic number equal to n ?"*. That leads us to a straightforward enumeration approach devoted to select n independent sets rather than assign colors to n^2 vertices. The branch and bound procedure is reinforced by an efficient filtering technique based on the cliques belonging to the not yet colored vertices

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of the graph. Thus, at each node of the tree, many independent sets are removed decreasing drastically the size of the remaining search space.

Experimentation carried out 3 new results:

χ(**Queens_**10^2)**=11** , χ(**Queens_**12^2)**=12** and χ(**Queens_**14^2)**=14**.

That constitutes the qualitative contribution of this work. Moreover the results for n=12 and n=14 are the same for Queens_$m \times n$ problems ($m \times n$ *chess boards for which* $m \leq n$) since the maximum clique number is still equal to n when $m < n$.

The figure bellow gives *certificates* for both Queens_12^2 and Queens_14^2 coloring numbers. Each letter represents a color.

A	B	C	D	E	F	G	H	I	J	K	L
L	G	K	I	H	C	J	E	D	B	F	A
E	C	F	L	D	K	B	I	A	G	J	H
H	J	I	B	F	L	A	G	K	D	C	E
K	D	A	G	J	H	E	C	F	L	I	B
C	E	L	F	B	D	I	K	G	A	H	J
J	A	G	K	I	E	H	D	B	F	L	C
I	K	E	C	A	G	F	L	J	H	B	D
D	F	H	J	L	B	K	A	C	E	G	I
G	L	D	E	K	J	C	B	H	I	A	F
F	H	B	A	C	I	D	J	L	K	E	G
B	I	J	H	G	A	L	F	E	C	D	K

A	B	C	D	E	F	G	H	I	J	K	L	M	N
F	J	M	B	N	C	D	L	K	G	I	H	E	A
E	G	L	K	D	B	M	I	H	N	J	A	C	F
J	I	H	M	C	E	F	N	A	D	L	B	K	G
N	D	K	L	G	A	J	C	B	H	E	I	F	M
G	F	N	C	M	I	E	A	J	L	B	K	D	H
I	E	A	H	L	K	N	G	F	M	D	C	B	J
M	H	D	E	F	G	B	K	L	I	A	J	N	C
K	L	B	J	I	M	H	D	C	F	N	G	A	E
B	A	G	F	K	D	C	J	N	E	H	M	L	I
C	M	E	I	J	H	L	B	D	A	F	N	G	K
H	K	F	G	A	N	I	M	E	B	C	D	J	L
L	C	I	N	B	J	A	F	G	K	M	E	H	D
D	N	J	A	H	L	K	E	M	C	G	F	I	B

Figure 1: Certificates for χ_{12}=12 and χ_{14}=14

References

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