Flooding Time in *Markov-Dynamic* Networks

Andrea **Clementi** (Speaker)
Claudio **Macci**
Francesco **Pasquale**

University of “**Tor Vergata**”
Rome

Angelo **Monti**
Riccardo **Silvestri**

University of “**La Sapienza**”
Rome

The “dynamic”-world challenge

MAIN RESEARCH GOAL:

Provide a *Mathematical Approach* for

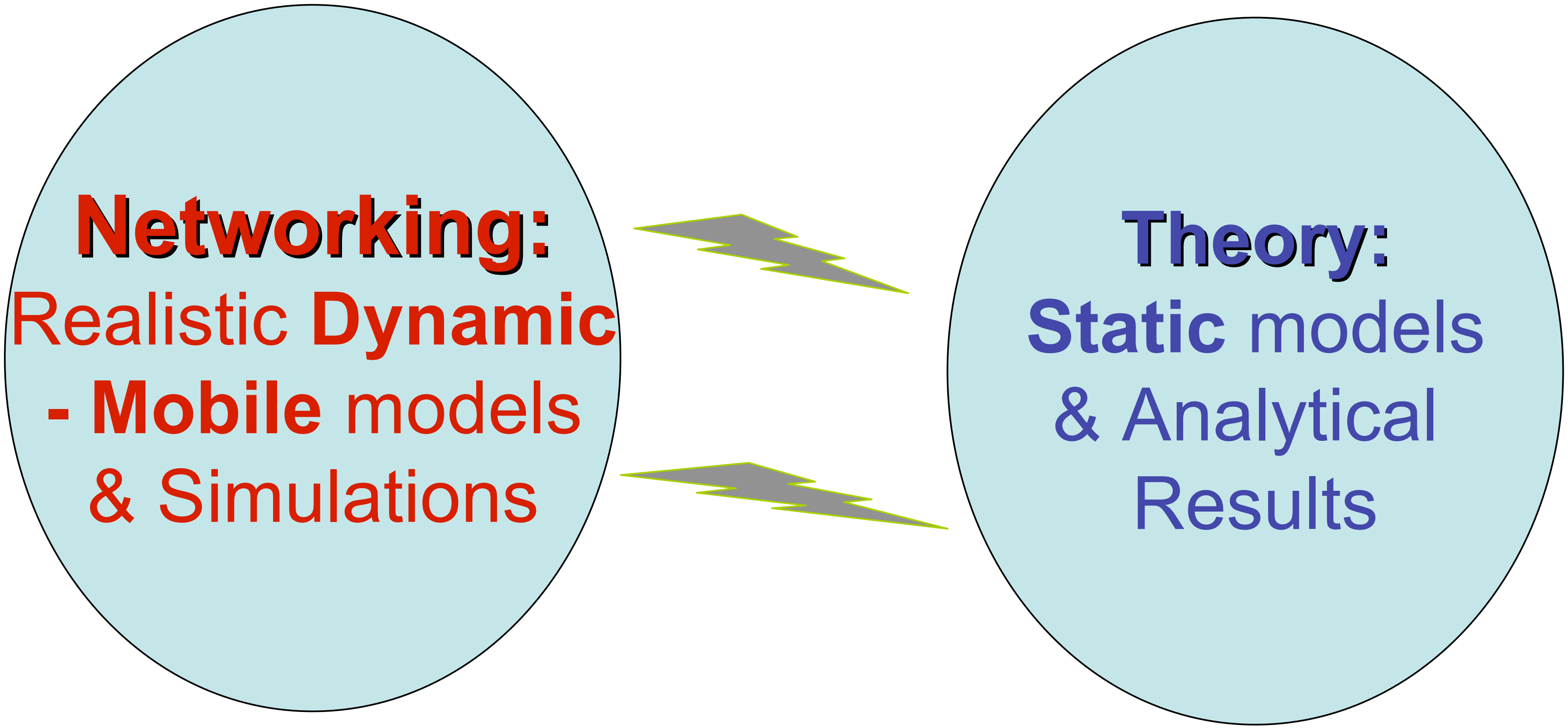
Information Spreading

on

EVOLVING (RADIO) NETWORKS

Practice *vs* **Theory**

- Scenario before **2001**



Dynamic Networks

A ***dynamic*** network is a sequence of graphs $G(V,E(t))$ ($V = [n]$) $t= 0,1,\ldots$ where

edge sets $E(t)$ may **change**

during the execution of a protocol

$E(0), E(1), \ldots, E(t), \ldots$

Dynamic Networks

The *Dynamic Network* model is very general: the edge-set sequence can be determined by any law/adversary:

- deterministic worst-case (Adversarial Networks)
- random oblivious changes
- node mobility (geometric models)

Theor. Research Advances (2001-…)

2008-..

Markov **Geometric** RND Changes

This Talk PODC'08

Markov Time-Depend. RND Changes

2007

Worst-c. Changes in Unkn. topology

Fully-Indep. RND Changes

2005-6

Worst-c. faults In Known topology

2001

Random Dynamic Networks

Previous **analytical** results on communication protocols concern dynamic random models assuming

Fully Spatial and Time Independence

over the **edge set**

Dynamic Networks

- **Goal:**

  a) Introduce ***Stocastic Time-Dependency*** on the edge-set sequence

  b) Provide **analytical** bounds on **Flooding Time**

Markov Dynamic Graphs

We consider the discrete version of the

Reciprocity-Graph Model [Wasserman 80]

Evolving Social Networks

Markov-Dynamic Networks

Let **$E(0)$** be any initial edge set and let **V^2** be the set of all possible (undirected) edges.

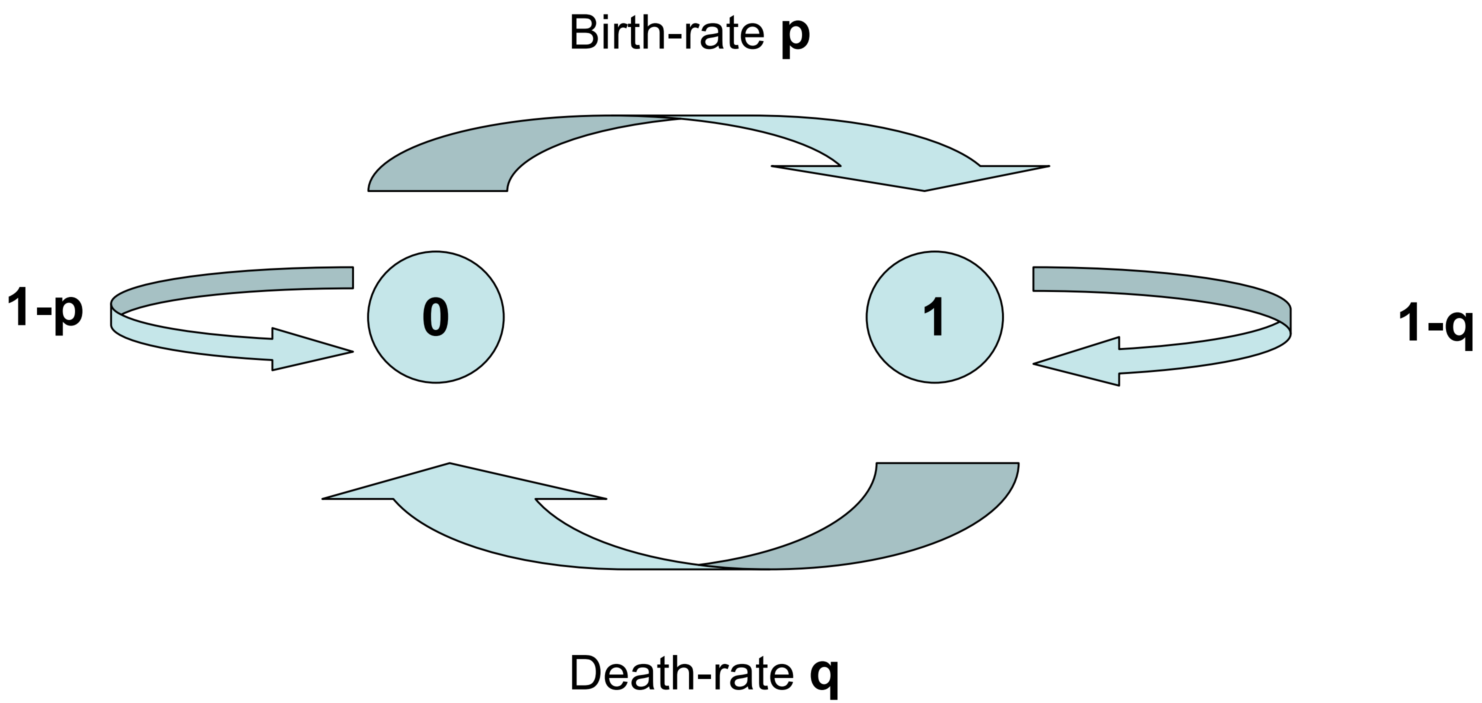
At every time step **$t \geq 1$**, the existence of any edge **$e \in V^2$** is determined by a ***Markov*** chain:

- If **$e \in E(t-1)$** then **$e \notin E(t)$** with probability **q**
- If **$e \notin E(t-1)$** then **$e \in E(t)$** with probability **p**

$q = q(n)$ is the edge ***death***-rate

$p = p(n)$ is the edge ***birth***-rate

Markov-Dynamic Graphs



Markov-Dynamic Graphs

Remarks.

- There is *independence* among edges

- There is Markov dependence in the time-sequence of every edge.

Markov-Dynamic Graphs

Average node degree in the *stationary* graph distribution

$$d(v) = (n-1)\,(p / (p+q))$$

- $p = k/n$ where $k = o(n)$ and $q = ½$ **then**

$$d(v) \approx k$$

- $p = q = k/n$ where $k = o(n)$ **then**

$$d(v) \approx n$$

So q seems to play a crucial role…..

Special Markov-Dynamic

(Independent) Dynamic-Random Graphs

$$p = 1-q$$

At every time step,

$G(V,E(t)) \approx G(n,p)$ (Random Graphs)

This is similar to the *Kleinberg et al* dynamic model where, at **every** time step, every node chooses a set of $k=p(n-1)$ neighbors, independently at random

Special Markov-Dynamic

Densifying networks: **$q = 0$**

An edge will never die.

Static Networks: **$p=q=0$**

Flooding

Broadcast Task: A source node **s** aims to send a message ***m*** to all nodes. A node is ***informed*** if it has received ***m***.

Flooding Mechanism: at every time step, every ***informed*** node sends ***m*** to all its neighbors.

Flooding

Flooding time is

the number of time steps required to ***inform*** *all nodes*.

Flooding

- In unknown, highly-dynamic networks, Flooding is often the “only” simple protocol for broadcasting.
- Flooding time is a natural **lower** bound: it used to evaluate other broadcast protocols
- Flooding in ***Dynamic*** Networks

  || || ||

  Diameter in ***Static*** Networks

Our Results

We determine tight bounds on **Flooding Time**

$T = T(n,p,q,E(0))$

on Markov-Dynamic Graphs,

for ***all*** values (functions) of **p** and **q**.

Flooding Time

Upper Bound. For any $0 \le p,q \le 1$ and **for any** $E(0)$,

$$T = O(\log n / \log(1+np))$$

Corollary. Let $p = \Omega(1/n)$ and any q, then Flooding Time is $O(\log n)$ starting from **any initial edge set**

Previous Related Results

- **fully-independent** random dynamic networks.

Flooding Time is $\mathbf{O(\mathit{log}\ n)}$ for $\mathbf{k = \Theta(1)}$ random neighbors.

[KempeKleinberg02],[Pittel87]

Same result holds in $\mathbf{G\ (n,p)}$ $(\mathbf{p = 1\text{-}q \geq 1/n})$

Flooding Time

Consequence 1:

Under very general assumptions (**$p \geq 1/n$, any q**)

Markovian time-dependency over the edge set **does not (asymtotically) decrease** the speed of data propagation in dynamic random graphs.

Flooding Time

Important Consequence 2:

(almost dense graphs)

When p **$\geq 1/n^{\delta}$** with fixed **$0 < \delta < 1$** and *any* **q**,

Flooding Time is **constant** and so

it ***does not depend*** on the initial topology.

Flooding Time

The Lower Bounds.

- **For any $0 \le p,q \le 1$ and starting conf. $E(0) = \circ$**

$$T = \Omega(\log n / np)$$

- **For any $p \ge \log n / n$, $0 \le q \le 1$ and $E(0) = \circ$**

$$T = \Omega(\log n / \log(1+np))$$

Flooding Time

Note. Flooding time can be ***slow***:

$p \approx 1/n^2$ then $T = \Theta(n \log n)$

Flooding Time

Matching Bounds.

Upper and Lower bounds are **tight** when

$p \leq 1/n$ ***OR*** $p \geq \log n / n$

for **any** $0 \leq q \leq 1$

Death-rate q may play a relevant role only in range

$1/n \leq p \leq \log n / n$

Flooding Time

Range: $1/n \le p \le \mathit{log}\ n / n$ (by varying q)

- $1/n \le p \le \mathit{loglog}\ n / n$

  $\mathit{log}\ n / \mathit{logloglog}\ n \le T \le \mathit{log}\ n$ (MAX)

- $\mathit{loglog}\ n / n \le p \le \mathit{log}\ n / n$

  $\mathit{log}\ n / \mathit{loglog}\ n \le T \le \mathit{log}\ n / \mathit{logloglog}\ n$

  (MIN)

Flooding Time: Densifying Networks

In range $1/n \leq p \leq \log n / n$ we consider case **q=0** (an edge never dies)

- $T = \Theta(\log n / np)$ when $p \leq \log\log n / n$
- $T = \Theta(\log n / \log\log n)$ when

  (plateau!) $\log\log n / n \leq p \leq \log n / n$

Upper Bound: Proof's Overview

Classic approach in ***static*** random graphs:

(Flooding = Diameter)

a) Prove (edge/vertex) ***expansion*** properties of the graph

b) Use such properties to evaluate the *height* of BFS Trees

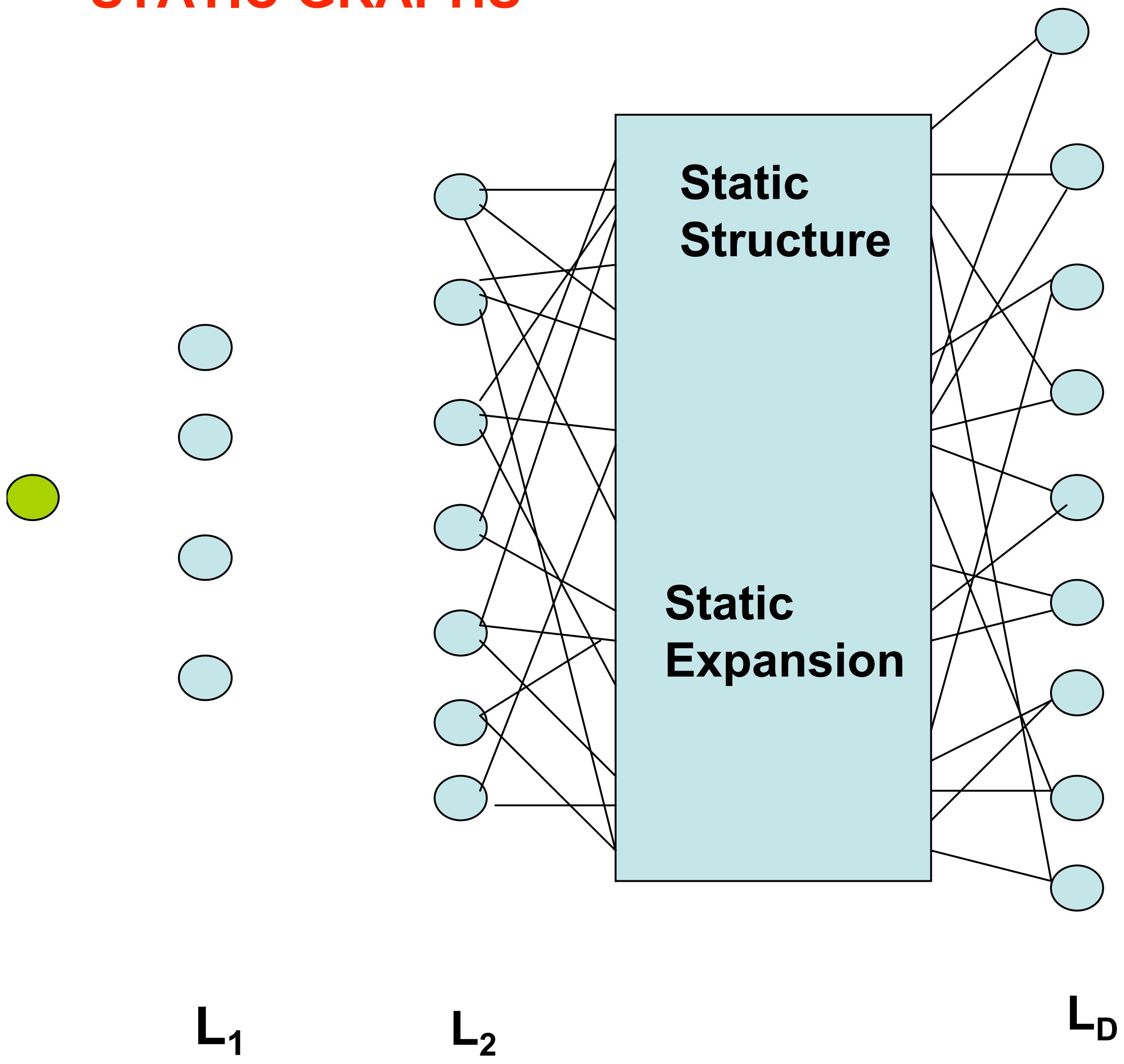
Upper Bound: Proof's Overview

In our dynamic world we

a) Define and prove **time/dynamic expansion** properties of a graph sequence

b) Use them to evaluate, at every step, the number of **new informed** nodes.

STATIC GRAPHS



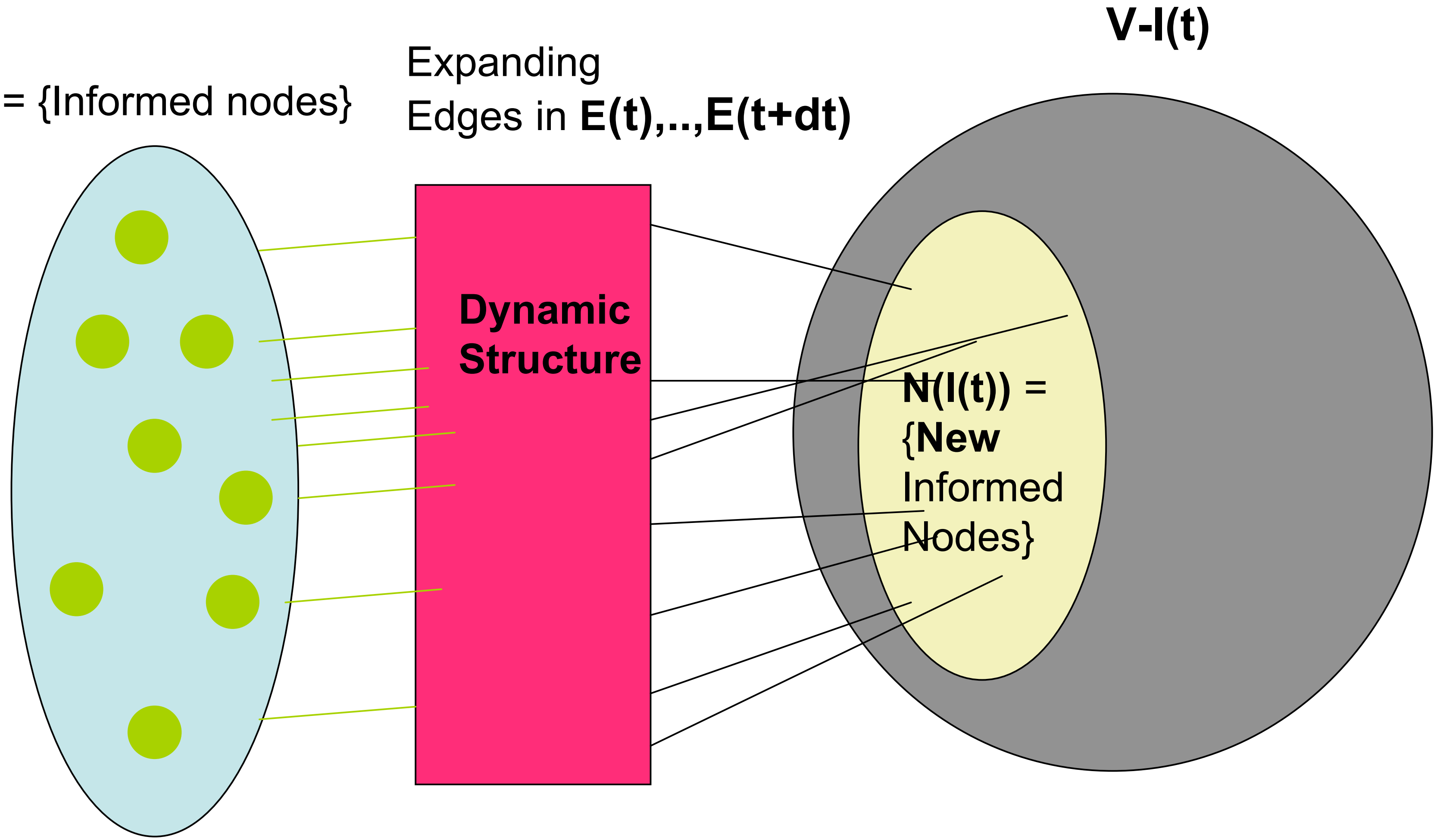
Apply **Static** Expansion between **fixed** Levels

Flooding Time

|||

Diameter D

Dynamic Graphs: Picture at time **t**



Conclusions

- We introduce a *Markovian-Dynamic Graph model*
- Derive a *dynamic* version of **expanding** properties of random graphs
- Apply them to analyze the **flooding** process

Future Work

"Immediate" technical questions:

- Tight bound in range **$1/n \le p \le \log n / n$**
- Starting from the ***Stationary Distribution***

Current (Challenging) Work

- Introduce ***spatial*** Markovian-dependency by means of

  simple/random node-mobility

- Adopt the same approach:

Dynamic expansion+ Flooding Phases