Controller Synthesis and Ordinal Automata

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ATVA 2006

Plan: Automata, Logics and Infinite Games

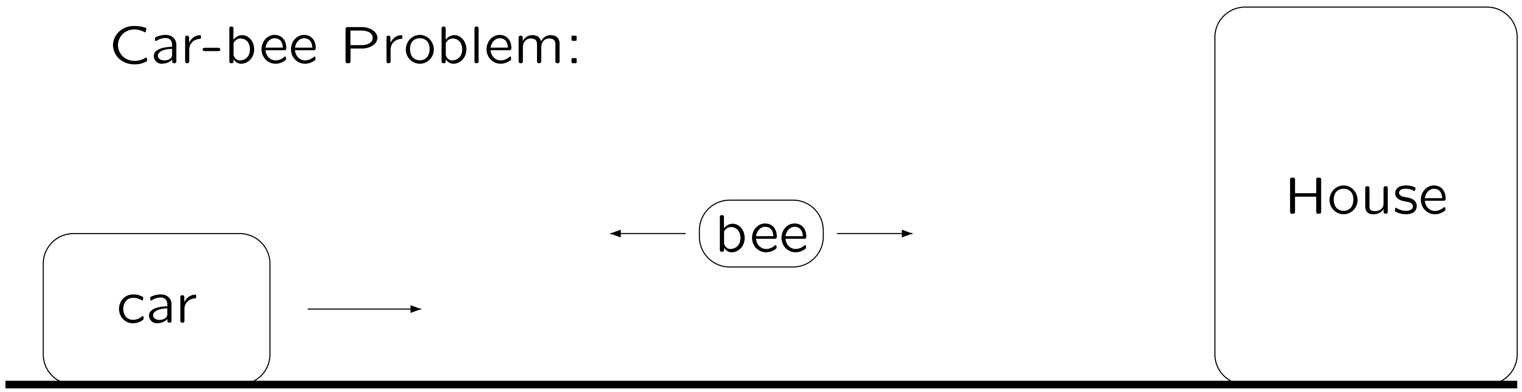
Abstract

Ordinal automata are used to model systems admitting Zeno behavior. Demri and Nowak introduced recently a logic LTL(ω^k) to reason about such systems. We consider a system with controllable/uncontrollable and observable/unobservable actions and a specification given by a formula of LTL(ω^k). The Problem is to construct a controller, modelled by a Büchi automaton, such that the product of the controller and the system satisfies the formula. We solve this problem using games of incomplete information.

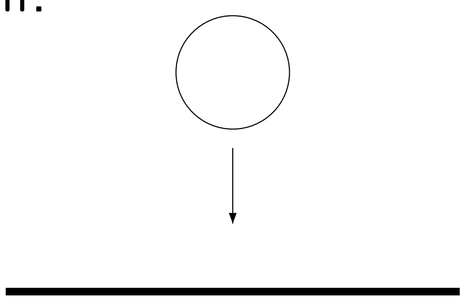
Motivation: Model Zeno behavior

Infinitely many actions in a finite time

Car-bee Problem:



Bouncing ball:



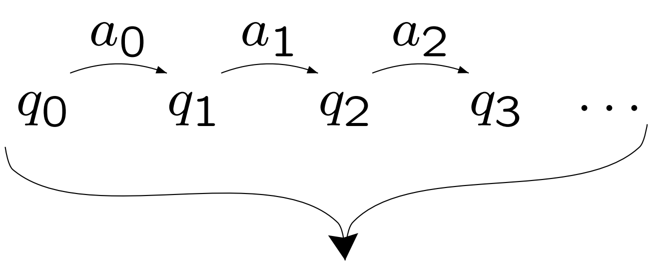
Physical system modeled
by an ordinal automaton...

Recall Muller Automata on ω-words

Finite state space Q, finite input alphabet Σ, initial state(s)

transition relation $\Delta \subseteq (Q \times \Sigma \times Q)$

run $r = (q_\beta)_{\beta \leq \omega}$ $\qquad \forall \beta < \omega,\ (q_\beta, a_\beta, q_{\beta+1}) \in \Delta$

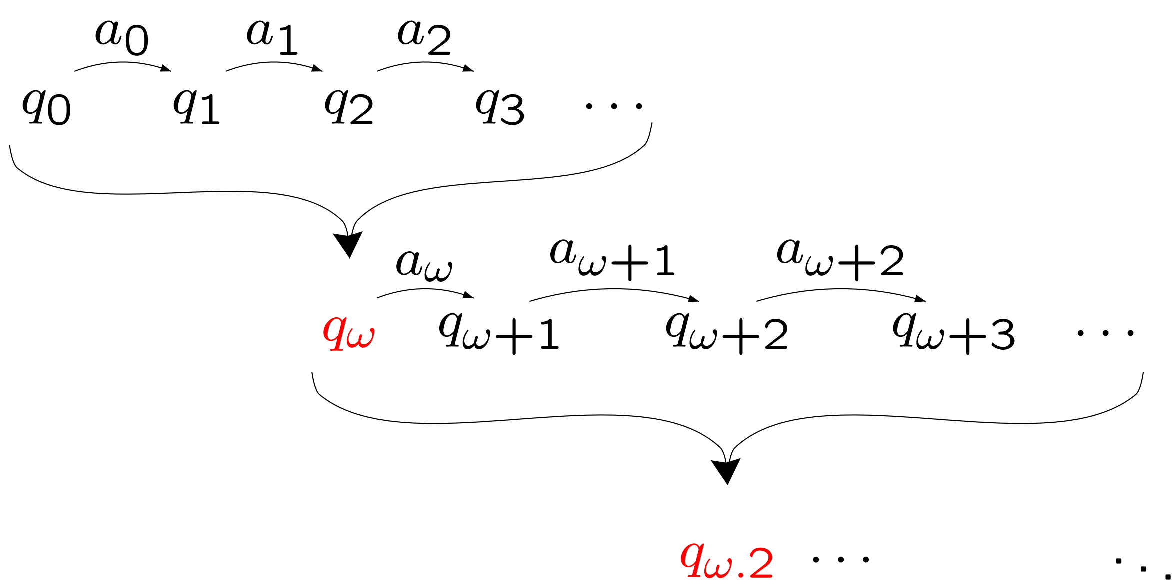


accepting or not, depending on states visited infinitely often:

$$Cofinal(\omega, r) = \{q \in Q : \forall \gamma < \omega, \exists\, \gamma < \gamma' < \omega, r_{\gamma'} = q\}$$

Ordinal Automata on ω^k [Büchi64], ... [CartonRispal 04]

Finite state space Q, finite input alphabet Σ, initial state(s)



transition relation $\Delta \subseteq (Q \times \Sigma \times Q) \cup (2^Q \times Q)$

run $r = (q_\beta)_{\beta \leq \omega^k}$ $\quad\quad$ $\forall \beta < \omega^k,\ (q_\beta, a_\beta, q_{\beta+1}) \in \Delta$

fora all limit ordinal β: $\quad\quad$ $(Cofinal(\beta, r), q_\beta) \in \Delta$

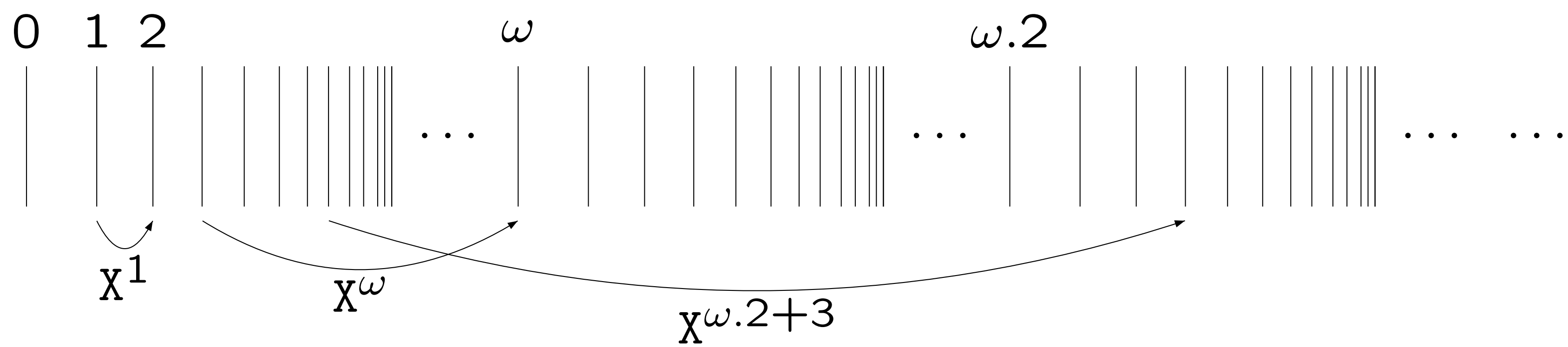
$Cofinal(\beta, r) = \{q \in Q : \forall \gamma < \beta, \exists\, \gamma < \gamma' < \beta, r_{\gamma'} = q\}$

Accepting if q_{ω^k} is final

Logic LTL(ω^k) [DemriNowak 05]

Atomic proposition, boolean connectives

in this slide $k = 2$



$\sigma, \beta \models \phi_1 \mathtt{U}^{\beta'} \phi_2$ iff there is $\gamma < \beta'$ such that $\sigma, \beta + \gamma \models \phi_2$ and for every $\gamma' < \gamma$, $\sigma, \beta + \gamma' \models \phi_1$.

standard abbreviations: $\mathtt{F}^\beta \phi = \top \mathtt{U}^\beta \phi$ and $\mathtt{G}^\beta \phi = \neg \mathtt{F}^\beta \neg \phi$

Model Checking [DemriNowak 05]

System $\mathscr{S}$ modeled by an ordinal (ω^k) automaton
formula $\phi \in \text{LTL}(\omega^k)$

does $\mathscr{S} \models \phi$?

decidable:
translation $\text{LTL}(\omega^k) \longrightarrow$ MSO (can be improved to FO)
+ [Büchi 64]

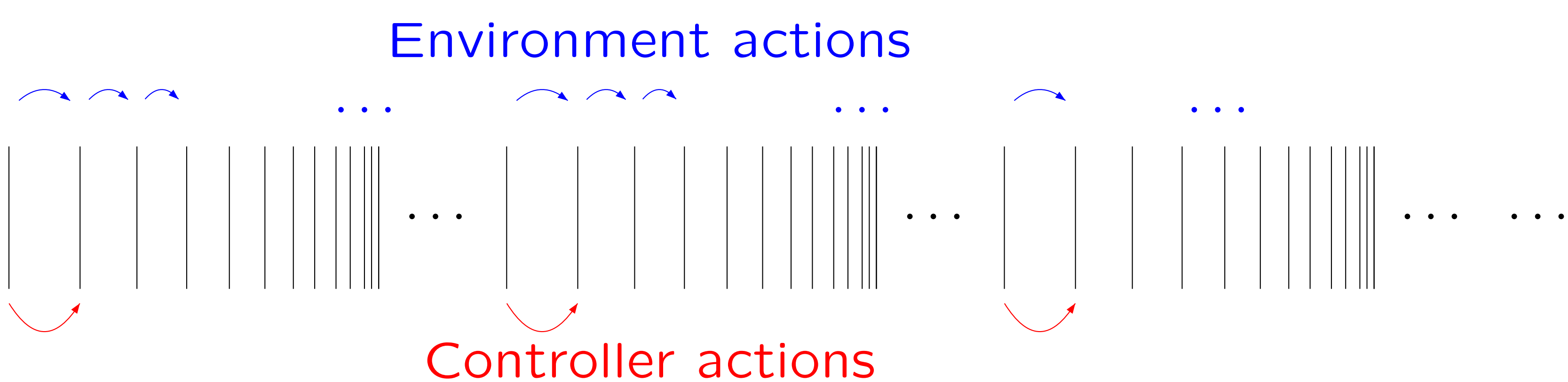
Using succinct ordinal automata,
Satisfiability of $\text{LTL}(\omega^k)$ is EXPSPACE-complete
Model-Checking for $\text{LTL}(\omega^k)$ is EXPSPACE-complete

Control Problem

(physical) System $\mathscr{S}$ modeled by an ordinal (ω^k) automaton
formula $\varphi \in \mathsf{LTL}(\omega^k)$

Is there a Controller $\mathscr{C}$ such that $\mathscr{S} \times \mathscr{C} \models \varphi$?

Controller must have usual properties regarding controllable/uncontrollable and observable/unobservable actions. Moreover Controller acts every ω^{k-1} steps of Environment:



Game between Controller and Environment

Arena = ordinal automaton $\mathscr{A} = \mathscr{S} \times \mathscr{A}_{\neg\varphi}$ with state space Q

Env tries to build an accepting run of $\mathscr{A}$,
Cont tries to prevent him

Starting in an initial state of $\mathscr{A}$,

1. Env chooses uncontrollable actions
2. Cont chooses controllable actions
   1.+2. $\rightsquigarrow$ one transition in $\mathscr{A}$
3. Env makes ω^{k-1} moves
4. Goto 1.

Summarizing ω^{k-1} steps

Pre-compute $\mathscr{R} \subseteq Q \times 2^Q \times Q$ such that

$(q, P, q') \in \mathscr{R} \iff \exists$ path $q \xrightarrow[\mathscr{A}]{\omega^{k-1}} q'$ where
set of states seen along this path is exactly P

$\rightsquigarrow$ (q, P, q') summarize ω^{k-1} moves of Env

the game is now:

1. $i = 0$ and q_0 is an initial state of $\mathscr{A}$.
2. Env chooses uncontrollable actions and then Cont chooses controllable actions $\rightsquigarrow$ one step $q_i \longrightarrow q_i'$
3. Env chooses $(q_i', P, q_{i+1}) \in \mathscr{R}$
4. $i = i + 1$, continue at point 2.

Incomplete information

Cont can not know exactly the current position of the game for several reasons:
- there are unobservable actions of Env
- Cont do not observe the ω^{k-1} steps (summarized in $\mathscr{R}$)
- $\mathscr{A}$ does not need to be deterministic

new game, where a position is a subset $Q_i \subseteq Q$ (as seen by Cont)

Play: $Q_0, Q'_0, Q_1, Q'_1, \cdots$

After the game is played, one has to find a concrete trace in the abstract play. Use a non deterministic Muller automaton

Solution of the Game

...Use a non deterministic Muller automaton
$->$ Transformation to non-deterministic Büchi automaton
$->$ Transformation to deterministic Rabin automaton
$->$ Transformation to deterministic parity automaton

$\rightsquigarrow$ parity game (on a finite graph). One can solve it

$\rightsquigarrow$ winning strategy is a (finite state) controller

cf [LNCS 2500]

Conclusion

Solution 2EXPTIME in the size of $\mathscr{A} = \mathscr{S} \times \mathscr{A}_{\neg\varphi}$.

Future work

Find a lower bound

Restrictions with better complexity

Extend to other linear ordering

Apply to timed automata